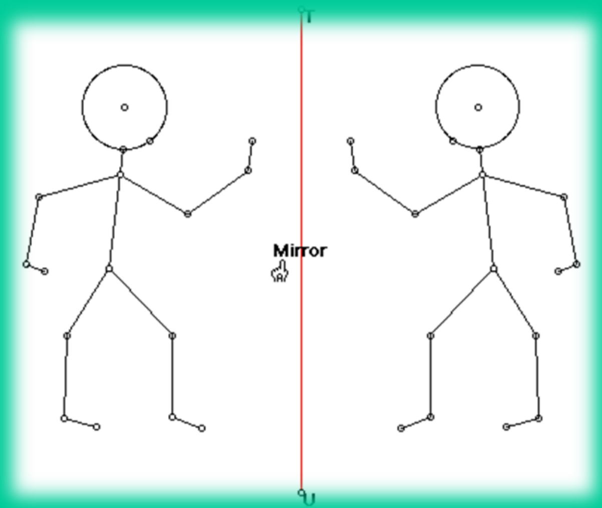
# Computer Animation Activity:

## Linking Matrices and Geometric Transformations

Presented By: Ashley Elmendorf

# Activity Significance



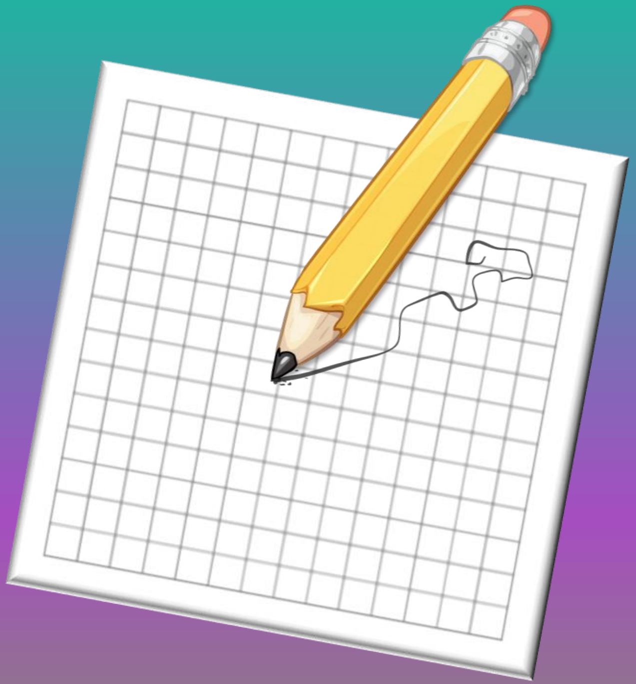

## Goals:

- Study a real-life application of mathematics
- Apply skills in
  - matrix multiplication
  - geometric transformation identification
- Explore connections between matrix multiplication and geometric transformations

## Objectives:

- Create matrices from the points that make up a figure drawn on the coordinate axis
- Perform matrix multiplication and plot the points in the product matrix
- Describe the geometric transformation that would change the figure in the same way as the matrix multiplication

# Preparation



**Materials:**

- Computer Animation Worksheet
- Pencils
- Graph Paper
- Straightedge

**Grade Level:**

- College
- Upper High School
- Lower Grade Simplifications

# Real-Life Application

Difference between early and modern animation creates appreciation for mathematics

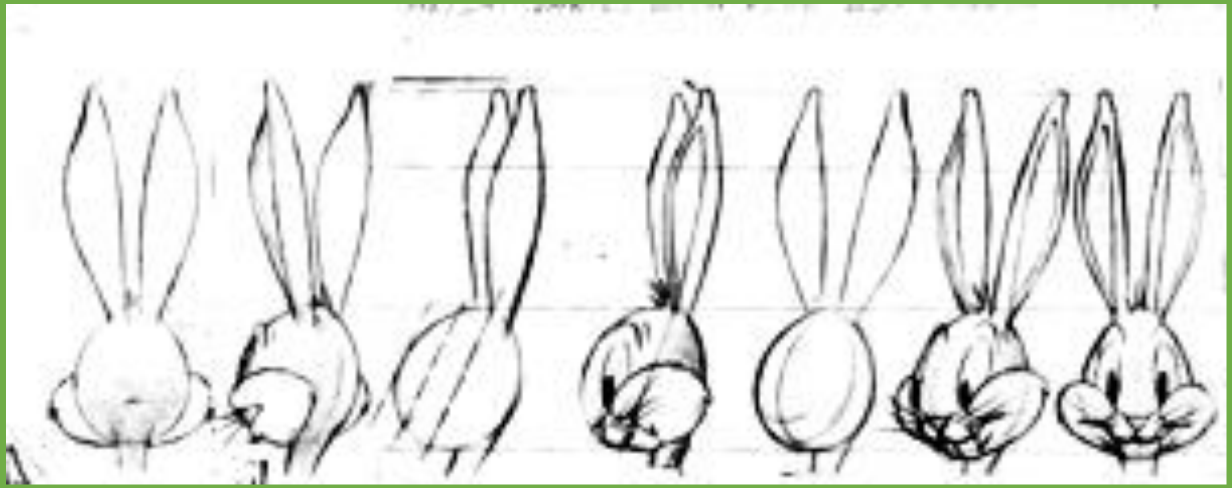
Relevant examples capture student interest

**Early Animation:**

thousands of hand-drawn images

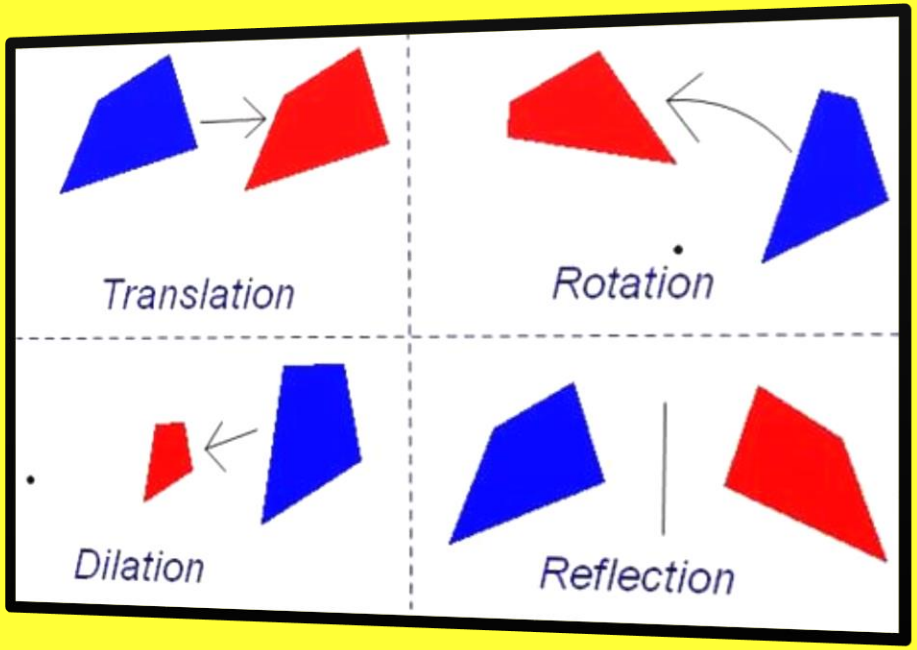
**Modern Animation:**

motion translated to computer language



# Transformations Background

- awhile since working with transformations
- practice during adolescent transformation lesson




**Definitions:**

- ***Translation*** – slides every point of a figure the same distance and direction
- ***Rotation*** – turns a figure about a fixed point
- ***Dilation*** – produces an image that is the same shape but a different size
- ***Reflection*** – creates an image on the opposite side of a line *or* builds a figure that is symmetric about a point

# Matrix Multiplication Background

- **Typically taught in upper-level college courses**
- **Matrices can be multiplied only if they have dimensions *A* = *m* x *n* and *B* = *n* x *p***
  - Yields *m* x *p* matrix
  - Matrix multiplication is not commutative
  - Multiply ***row*** of *A* by ***column*** of *B*

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \; x \; \begin{bmatrix} e & f & g \\ h & i & j \end{bmatrix} = \begin{bmatrix} ae+bh & af+bi & ag+bj \\ ce+dh & cf+di & cg+dj \end{bmatrix}$$

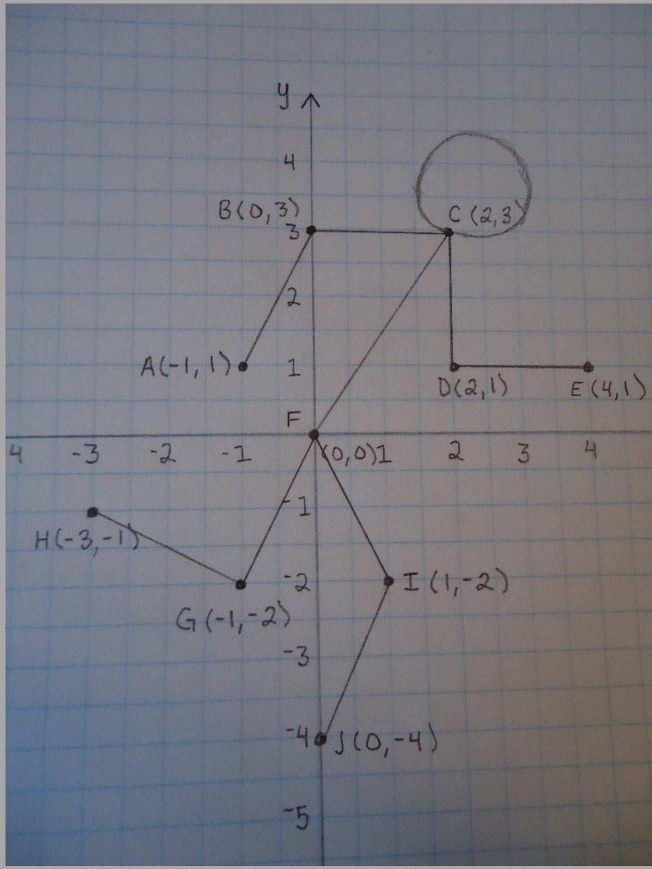
# Activity Procedure

**Step 1:** Distribute worksheet featuring labeled figure to be transformed.

*Note: Give unlabeled figure to younger students for plotting practice.*

**Step 2:** Instruct students to create *S*, a 2 x *z* matrix, from ordered pairs

- *z* = number of points in figure
- rows = *x* - values
- columns = *y* - values




$$S = \begin{bmatrix} -1 & 0 & 2 & 2 & 4 & 0 & -1 & -3 & 1 & 0 \\ 1 & 3 & 3 & 1 & 1 & 0 & -2 & -1 & -2 & -4 \end{bmatrix}$$

# Activity Procedure continued

**Step 3:** Students should complete parts *a* and *b* of questions 1 - 3.

a. multiply S by the given matrix
b. plot and connect the points in the product matrix (figure revealed)

*Note: Give product matrices to younger class.*
*Inform class that each column represents an ordered pair.*

**Step 4:** Ask students to answer part *c* of questions 1 - 3.

- Identify the transformation that produced each image.
- Draw any centers of rotation, lines and points of reflection, etc.

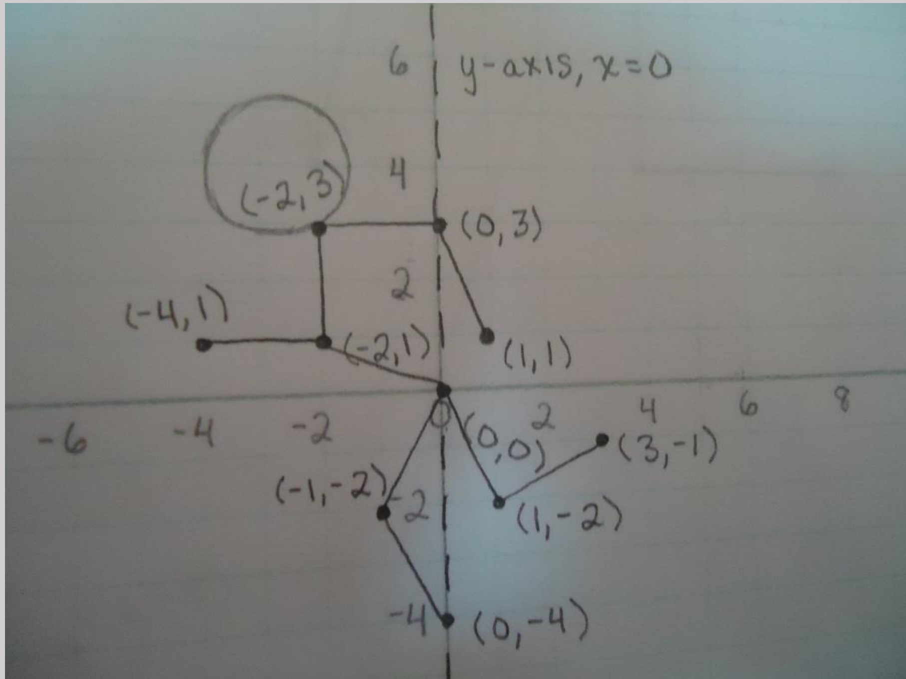
# Activity Procedure continued

**Question 1:** $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} x \begin{bmatrix} -1 & 0 & 2 & 2 & 4 & 0 & -1 & -3 & 1 & 0 \\ 1 & 3 & 3 & 1 & 1 & 0 & -2 & -1 & -2 & -4 \end{bmatrix}$

**Solution:** $\begin{bmatrix} -1x + 0y & \cdots \\ 0x + 1y & \cdots \end{bmatrix}$

- Negate $x$ - values
- Keep $y$ - values

**Transformation:** *reflection* over $y$ - axis



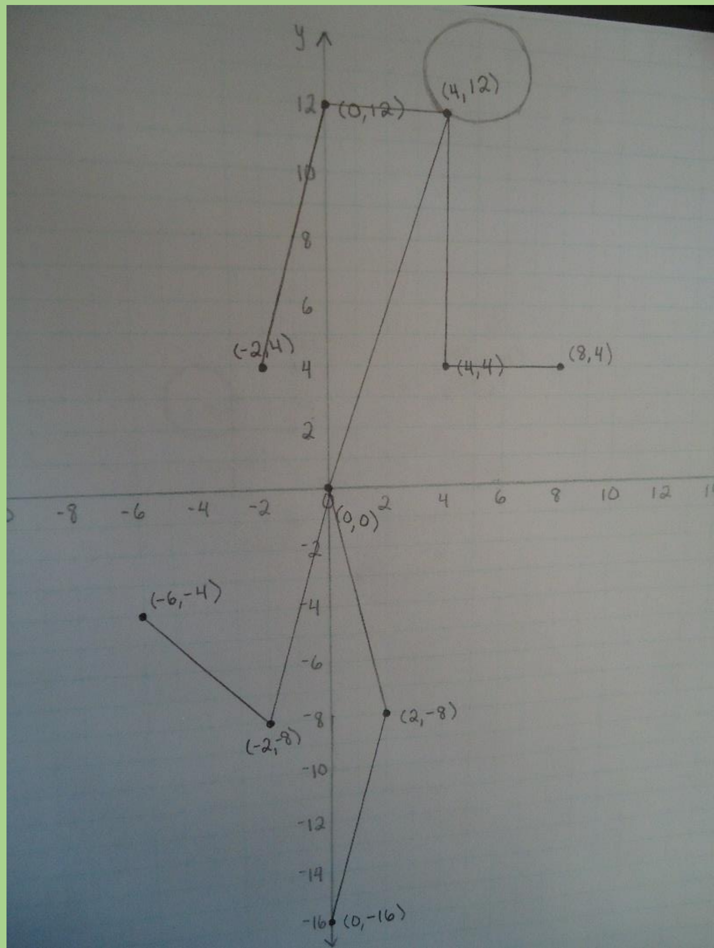
# Activity Procedure continued

**Question 2:** $\begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix} x \begin{bmatrix} -1 & 0 & 2 & 2 & 4 & 0 & -1 & -3 & 1 & 0 \\ 1 & 3 & 3 & 1 & 1 & 0 & -2 & -1 & -2 & -4 \end{bmatrix}$

**Solution:** $\begin{bmatrix} 2x + 0y & \cdots \\ 0x + 4y & \cdots \end{bmatrix}$

- Double $x$ - values
- Quadruple $y$ - values

**Transformation:** *dilation*: twice as wide and 4 times as tall



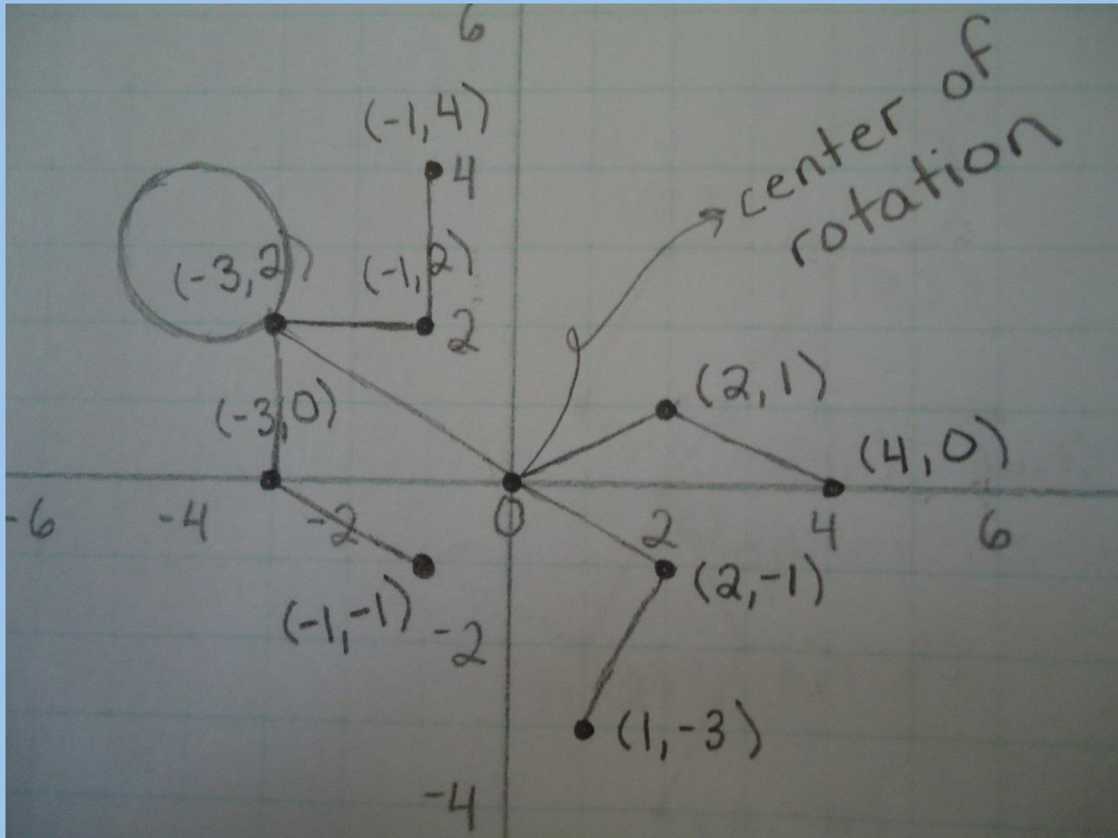
# Activity Procedure continued

**Question 3:** $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} x \begin{bmatrix} -1 & 0 & 2 & 2 & 4 & 0 & -1 & -3 & 1 & 0 \\ 1 & 3 & 3 & 1 & 1 & 0 & -2 & -1 & -2 & -4 \end{bmatrix}$

**Solution:** $\begin{bmatrix} 0x + -1y & \cdots \\ 1x + 0y & \cdots \end{bmatrix}$

- *x* – values become negated y - values
- *y* – values become x - values

**Transformation:** 90° *rotation* about (0, 0)



# Activity Procedure continued

**Step 5:** Assign question 4.

- Create R by adding a third row of 1's to S.
- Ignore the third row of the product matrix when plotting.

$$R = \begin{bmatrix} -1 & 0 & 2 & 2 & 4 & 0 & -1 & -3 & 1 & 0 \\ 1 & 3 & 3 & 1 & 1 & 0 & -2 & -1 & -2 & -4 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

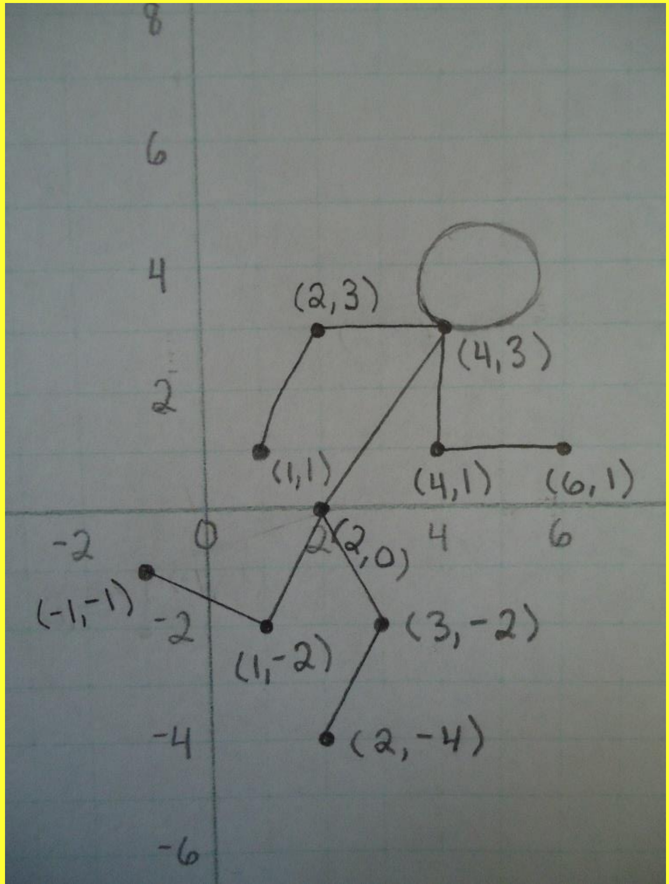
# Activity Procedure continued

**Question 4:** $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} x \begin{bmatrix} -1 & 0 & 2 & 2 & 4 & 0 & -1 & -3 & 1 & 0 \\ 1 & 3 & 3 & 1 & 1 & 0 & -2 & -1 & -2 & -4 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$

**Solution:** $\begin{bmatrix} 1x & + & 0y & + & 2 & \cdots \\ 0x & + & 1y & + & 0 & \cdots \\ 0x & + & 0y & + & 1 & \cdots \end{bmatrix}$

- Increase x - value by 2
- Keep y - value
- Keep third row of 1's

**Transformation:** *translation* 2 units to the right



# Some Alterations

- **Assign different figures**
  - add interest
  - encourage individual effort

- **Divide students into groups based on figure**
  - diversity
  - reduce number of assignments to grade

- **Ask students to work with favorite animated character for homework**

# Sources

*Geometry*. (1998-2012). Retrieved from Oswego City School District Regents Exam Prep Center: http://www.regentsprep.org/Regents/math/geometry/mathGEOMETRY.htm#m5

Russell, C. (2000-2013). *Computer Animation.* Retrieved from NCTM Illuminations: http://illuminations.nctm.org/LessonDetail.aspx?id=L841